WINTER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any FIVE of following:**	**10**
	a)	Define odd and even function with suitable example.	**02**
	Ans	If $f(-x)=-f(x)$ then the function is an odd function	½
		e.g. $f(x)=x^3+x$	
		$\therefore f(-x)=(-x)^3+(-x)$	
		$=-(x^3+x)$	
		$=-f(x)$	½
		If $f(-x)=f(x)$ then the function is an even function	½
		e.g. $f(x)=x^2+1$	
		$\therefore f(-x)=(-x)^2+1$	
		$=x^2+1$	
		$=f(x)$	½
		--	
	b)	If $f(x)=\dfrac{x^2+9}{\sqrt{x-3}}$, find $f(4)+f(5)$.	**02**
	Ans	$f(4)+f(5)=\left(\dfrac{4^2+9}{\sqrt{4-3}}\right)+\left(\dfrac{5^2+9}{\sqrt{5-3}}\right)$	½+½
		$=25+\dfrac{34}{\sqrt{2}}=49.042$	1
		OR	
		$f(4)=\dfrac{4^2+9}{\sqrt{4-3}}=25$	½

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1.	b)	$f(5)=\dfrac{5^2+9}{\sqrt{5-3}}=\dfrac{34}{\sqrt{2}}$	½
		$\therefore f(4)+f(5)$	
		$=25+\dfrac{34}{\sqrt{2}}$	1
		$=49.042$	
		--	
	c)	Find $\dfrac{dy}{dx}$ if $y=(3a)^x+x^{(\log 3)}+x^a+a^a$	**02**
	Ans	$\therefore \dfrac{dy}{dx}=(3a)^x\log 3a+\log 3\cdot x^{(\log 3)-1}+a\cdot x^{a-1}+0$	½+½+½+½
		$=(3a)^x\log 3a+\log 3\cdot x^{(\log 3)-1}+a\cdot x^{a-1}$	
		OR	
		$\because y=(3a)^x+x^{(\log 3)}+x^a+a^a$	
		$\because y=3^x a^x+x^{(\log 3)}+x^a+a^a$	
		$\therefore \dfrac{dy}{dx}=3^x a^x\log a+a^x 3^x\log 3+\log 3\cdot x^{(\log 3)-1}+a\cdot x^{a-1}+0$	½+½+½+½
		$=3^x a^x(\log a+\log 3)+\log 3\cdot x^{(\log 3)-1}+a\cdot x^{a-1}$	
		--	
	d)	Evaluate $\int x^2\cdot\log x\,dx$	**02**
	Ans	$\int x^2\cdot\log x\,dx=\log x\int x^2dx-\int\left[\int x^2dx\cdot\dfrac{d}{dx}\log x\right]dx$	½
		$=\log x\cdot\dfrac{x^3}{3}-\int\dfrac{x^3}{3}\cdot\dfrac{1}{x}dx$	½
		$=\log x\cdot\dfrac{x^3}{3}-\dfrac{1}{3}\int x^2dx$	
		$=\log x\cdot\dfrac{x^3}{3}-\dfrac{1}{3}\cdot\dfrac{x^3}{3}+c$	
		$=\log x\cdot\dfrac{x^3}{3}-\dfrac{x^3}{9}+c$	1
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	e)	Evaluate $\int \frac{dx}{x^2+4x+5}$	**02**
	Ans	$\int \frac{dx}{x^2+4x+5}$	
		Third term $=\frac{(4x)^2}{4\times x^2}=4$	½
		$=\int \frac{dx}{x^2+4x+4-4+5}$	½
		$=\int \frac{dx}{(x+2)^2+1}$	½
		$=\frac{1}{1}\tan^{-1}\left(\frac{x+2}{1}\right)+c$	
		$=\tan^{-1}(x+2)+c$	½
	f)	Find the area bounded by the curve $y=\sin x, x-$axis and the ordinate $x=0, x=\frac{\pi}{2}$	**02**
	Ans	Area $A=\int_a^b ydx$	
		$=\int_0^{\frac{\pi}{2}} \sin xdx$	1
		$=[-\cos x]_0^{\pi/2}$	½
		$=-[0-1]$	
		$=1$	½
	g)	State the trapezoidal rule of numerical integration.	**02**
	Ans	Trapezoidal rule	
		$\int_a^b f(x)dx=\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$	2
		where $h=\frac{b-a}{n}$	
2.		**Attempt any THREE of the following:**	**12**
	a)	Find $\frac{dy}{dx}$ if $x^2+y^2+xy-y=0$ at $(1,2)$	**04**

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Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	a)Ans	$\because x^2+y^2+xy-y=0$	
		$\therefore 2x+2y\frac{dy}{dx}+x\frac{dy}{dx}+y-\frac{dy}{dx}=0$	1
		$\therefore 2x+y+(2y+x-1)\frac{dy}{dx}=0$	
		$\therefore (2y+x-1)\frac{dy}{dx}=-(2x+y)$	1
		$\therefore \frac{dy}{dx}=\frac{-(2x+y)}{2y+x-1}$	1
		at $(1,2)$	
		$\frac{dy}{dx}=\frac{-(2(1)+2)}{2(2)+1-1}=-1$	1
		---	
	b)	If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$	**04**
	Ans	$\because x=a(\cos t+t\sin t)$	
		$\therefore \frac{dx}{dt}=a(-\sin t+t\cos t+\sin t)$	
		$=at\cos t$	1
		$\because y=a(\sin t-t\cos t)$	
		$\therefore \frac{dy}{dt}=a(\cos t+t\sin t-\cos t)$	
		$=at\sin t$	1
		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	
		$\therefore \frac{dy}{dx}=\frac{at\sin t}{at\cos t}$	
		$=\tan t$	1
		at $t=\frac{\pi}{4}$	
		$\frac{dy}{dx}=\tan\frac{\pi}{4}$	
		$=1$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c)	The rate of working of an engine is given by the expression $10V+\frac{4000}{V}$, where 'V' is the speed of the engine.Find the speed at which the rate of working is the least.	**04**
	Ans	The rate of working is, $W=10V+\frac{4000}{V}$	
		$\therefore\frac{dW}{dV}=10-\frac{4000}{V^2}$	½
		$\therefore\frac{d^2W}{dV^2}=\frac{8000}{V^3}$	½
		Consider $\frac{dW}{dV}=0$	½
		$\therefore 10-\frac{4000}{V^2}=0$	
		$\therefore 10=\frac{4000}{V^2}$	
		$\therefore V^2=400$	
		$\therefore V=20,-20$	1
		at $V=20$	
		$\therefore\frac{d^2W}{dV^2}=\frac{8000}{(20)^3}=1>0$	1
		$\therefore$ The speed is $V=20$ at which the rate of working is least	½
		--	
	d)	A telegraph wire hangs in the form of a curve $y=a\cdot\log\left[\sec\left(\frac{x}{a}\right)\right]$. Where '$a$' is constant.Show that the curvature at any point is $\frac{1}{a}\cos\left(\frac{x}{a}\right)$.	**04**
	Ans	$y=a\log\left(\sec\left(\frac{x}{a}\right)\right)$	
		$\therefore\frac{dy}{dx}=a\frac{1}{\sec\left(\frac{x}{a}\right)}\sec\left(\frac{x}{a}\right)\tan\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)$	
		$\therefore\frac{dy}{dx}=\tan\left(\frac{x}{a}\right)$	1
		$\therefore\frac{d^2y}{dx^2}=\sec^2\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)$	1
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	d)	$\therefore \rho = \dfrac{\left[1+\tan^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)}$	1
		$\therefore \rho = \dfrac{a\left[\sec^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)}$	
		$\therefore \rho = \dfrac{a\sec^3\left(\frac{x}{a}\right)}{\sec^2\left(\frac{x}{a}\right)}$	
		$\therefore \rho = a\sec\left(\frac{x}{a}\right)$	
		$\therefore \text{curvature} = \dfrac{1}{\rho} = \dfrac{1}{a}\cos\left(\frac{x}{a}\right)$	1
		--	
3.		**Attempt any THREE of the following:**	**12**
	a)	Find equation of tangent to curve $x=\frac{1}{t}, y=1-\frac{1}{t}$ when $t=2$.	**04**
	Ans	$x=\frac{1}{t}$ $\quad$ $y=1-\frac{1}{t}$	
		$\therefore \frac{dx}{dt}=\frac{-1}{t^2}$ $\quad$ $\frac{dy}{dt}=\frac{1}{t^2}$	½+½
		$\therefore \dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$	
		$=\dfrac{\frac{1}{t^2}}{\frac{-1}{t^2}}$	
		$=-1$	
		$\therefore$ slope of tangent $=m=-1$	1
		when $t=2$	
		$x=\frac{1}{t}=\frac{1}{2}$ $\quad$ $y=1-\frac{1}{t}=1-\frac{1}{2}=\frac{1}{2}$	

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3.	a)	$\therefore$ Point is $\left(\frac{1}{2},\frac{1}{2}\right)$	1
		$\therefore$ Equation of tangent is,	
		$y-\frac{1}{2}=-1\left(x-\frac{1}{2}\right)$	
		$\therefore y-\frac{1}{2}=-x+\frac{1}{2}$	
		$\therefore x+y-1=0$	1
		--	
	b)	Find $\frac{dy}{dx}$ if $y=x^x+x^{\sqrt{x}}$	**04**
	Ans	Let $u=x^x$	
		$\therefore \log u=\log x^x$	½
		$=x\log x$	
		$\therefore \frac{1}{u}\frac{du}{dx}=x\cdot\frac{1}{x}+\log x$	
		$=1+\log x$	
		$\therefore \frac{du}{dx}=u\left(1+\log x\right)$	
		$=x^x\left(1+\log x\right)$	1
		Let $v=x^{\sqrt{x}}$	
		$\therefore \log v=\log x^{\sqrt{x}}$	½
		$=\sqrt{x}\log x$	
		$\therefore \frac{1}{v}\frac{dv}{dx}=\sqrt{x}\cdot\frac{1}{x}+\log x\cdot\frac{1}{2\sqrt{x}}$	
		$=\frac{1}{\sqrt{x}}+\frac{\log x}{2\sqrt{x}}$	
		$\therefore \frac{dv}{dx}=v\left(\frac{1}{\sqrt{x}}+\frac{\log x}{2\sqrt{x}}\right)$	
		$=x^{\sqrt{x}}\left(\frac{1}{\sqrt{x}}+\frac{\log x}{2\sqrt{x}}\right)$	1
		$\because y=u+v$	
		$\therefore \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$	
		$=x^x\left(1+\log x\right)+x^{\sqrt{x}}\left(\frac{1}{\sqrt{x}}+\frac{\log x}{2\sqrt{x}}\right)$	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left[\frac{x}{\sqrt{1-x^2}}\right]$	**04**
	Ans	$y=\tan^{-1}\left[\frac{x}{\sqrt{1-x^2}}\right]$	
		Let $x=\sin\theta \quad \therefore \theta=\sin^{-1}x$	1
		$\therefore y=\tan^{-1}\left[\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right]$	
		$=\tan^{-1}(\tan\theta)$	1
		$=\theta$	1
		$=\sin^{-1}x$	
		$\therefore \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$	1
		OR	
		Let $x=\cos\theta \quad \therefore \theta=\cos^{-1}x$	1
		$\therefore y=\tan^{-1}\left[\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}\right]$	
		$=\tan^{-1}(\cot\theta)$	1
		$=\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\theta\right)\right)$	½
		$=\frac{\pi}{2}-\theta$	½
		$=\frac{\pi}{2}-\cos^{-1}x$	
		$\therefore \frac{dy}{dx}=-\frac{-1}{\sqrt{1-x^2}}$	
		$=\frac{1}{\sqrt{1-x^2}}$	1
		OR	
		$y=\tan^{-1}\frac{x}{\sqrt{1-x^2}}$	
		$\therefore \frac{dy}{dx}=\frac{1}{1+\left(\frac{x}{\sqrt{1-x^2}}\right)^2}\cdot\frac{\sqrt{1-x^2}(1)-x\cdot\frac{1}{2\sqrt{1-x^2}}(-2x)}{1-x^2}$	2

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	$=\dfrac{1-x^2}{1-x^2+x^2}\cdot\dfrac{\sqrt{1-x^2}+\dfrac{x^2}{\sqrt{1-x^2}}}{1-x^2}$	
		$=\sqrt{1-x^2}+\dfrac{x^2}{\sqrt{1-x^2}}$	1
		$=\dfrac{1-x^2+x^2}{\sqrt{1-x^2}}$	
		$=\dfrac{1}{\sqrt{1-x^2}}$	1
		---	
	d)	Evaluate $\int\dfrac{\sec^2 x}{(1+\tan x)(3+\tan x)}dx$	**04**
	Ans	$\int\dfrac{\sec^2 x}{(1+\tan x)(3+\tan x)}dx$	
		Let $\tan x=t$	
		$\therefore \sec^2 xdx=dt$	1
		$=\int\dfrac{1}{(1+t)(3+t)}dt$	
		Consider	
		$\dfrac{1}{(1+t)(3+t)}=\dfrac{A}{1+t}+\dfrac{B}{3+t}$	½
		$\therefore 1=A(3+t)+B(1+t)$	
		Put $t=-1$ $\therefore A=\dfrac{1}{2}$	½
		Put $t=-3$ $\therefore B=\dfrac{-1}{2}$	½
		$\therefore\dfrac{1}{(1+t)(3+t)}=\dfrac{\frac{1}{2}}{1+t}+\dfrac{-\frac{1}{2}}{3+t}$	
		$\therefore\int\dfrac{1}{(1+t)(3+t)}dt=\int\left(\dfrac{\frac{1}{2}}{1+t}+\dfrac{-\frac{1}{2}}{3+t}\right)dt$	
		$=\dfrac{1}{2}\log(1+t)-\dfrac{1}{2}\log(3+t)+c$	½+½
		$=\dfrac{1}{2}\log\left(\dfrac{1+t}{3+t}\right)+c$	
		$=\dfrac{1}{2}\log\left(\dfrac{1+\tan x}{3+\tan x}\right)+c$	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate : $\int \frac{1}{x\left[9+\left(\log_e x\right)^2\right]}dx$	**04**
	Ans	$\int \frac{1}{x\left[9+\left(\log_e x\right)^2\right]}dx$	
		Let $\log_e x = t$	
		$\therefore \frac{1}{x}dx = dt$	1
		$= \int \frac{1}{9+t^2}dt$	
		$= \int \frac{1}{3^2+t^2}dt$	1
		$= \frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)+c$	1
		$= \frac{1}{3}\tan^{-1}\left(\frac{\log_e x}{3}\right)+c$	1
		--	
	b)	Evaluate: $\int \frac{1}{2\sin x+3\cos x}\,dx$	**04**
	Ans	$\int \frac{1}{2\sin x+3\cos x}\,dx$	
		Let $\tan\frac{x}{2}=t$	1
		$\therefore \sin x=\frac{2t}{1+t^2}, \cos x=\frac{1-t^2}{1+t^2}, dx=\frac{2dt}{1+t^2}$	
		$= \int \frac{1}{2\left(\frac{2t}{1+t^2}\right)+3\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$	
		$= \int \frac{1}{4t+3-3t^2}\cdot 2dt$	
		$= \int \frac{1}{-\left(3t^2-4t-3\right)}\cdot 2dt$	½
		Third term $=\frac{\left(-4t\right)^2}{4\times 3t^2}=\frac{4}{3}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$=-2\int\frac{1}{3t^2-4t+\frac{4}{3}-\frac{4}{3}-3}dt$	
		$=-2\int\frac{1}{\left(\sqrt{3}t-\frac{2}{\sqrt{3}}\right)^2-\left(\sqrt{\frac{13}{3}}\right)^2}dt$	½
		$=-2\cdot\frac{1}{2\sqrt{\frac{13}{3}}}\log\left(\frac{\sqrt{3}t-\frac{2}{\sqrt{3}}-\sqrt{\frac{13}{3}}}{\sqrt{3}t-\frac{2}{\sqrt{3}}+\sqrt{\frac{13}{3}}}\right)\cdot\frac{1}{\sqrt{3}}+c$	1
		$=\frac{-1}{\sqrt{13}}\log\left(\frac{\sqrt{3}\tan\frac{x}{2}-\frac{2}{\sqrt{3}}-\sqrt{\frac{13}{3}}}{\sqrt{3}\tan\frac{x}{2}-\frac{2}{\sqrt{3}}+\sqrt{\frac{13}{3}}}\right)+c$	½
		$=\frac{-1}{\sqrt{13}}\log\left(\frac{3\tan\frac{x}{2}-2-\sqrt{13}}{3\tan\frac{x}{2}-2+\sqrt{13}}\right)+c$	
		OR	
		Let $\tan\frac{x}{2}=t$	
		$\therefore\sin x=\frac{2t}{1+t^2},\cos x=\frac{1-t^2}{1+t^2},dx=\frac{2dt}{1+t^2}$	1
		$=\int\frac{1}{2\left(\frac{2t}{1+t^2}\right)+3\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$	
		$=\int\frac{1}{4t+3-3t^2}\cdot 2dt$	
		$=\int\frac{1}{-3\left(t^2-\frac{4}{3}t-1\right)}\cdot 2dt$	½
		$=\frac{-2}{3}\int\frac{1}{t^2-\frac{4}{3}t-1}dt$	
		Third term $=\frac{\left(\frac{-4}{3}t\right)^2}{4t^2}=\frac{4}{9}$	½

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4.	b)	$=\frac{-2}{3}\int\frac{1}{t^2-\frac{4}{3}t+\frac{4}{9}-\frac{4}{9}-1}dt$	
		$=\frac{-2}{3}\int\frac{1}{\left(t-\frac{2}{3}\right)^2-\left(\frac{\sqrt{13}}{3}\right)^2}dt$	½
		$=\frac{-2}{3}\cdot\frac{1}{2\frac{\sqrt{13}}{3}}\log\left(\frac{t-\frac{2}{3}-\frac{\sqrt{13}}{3}}{t-\frac{2}{3}+\frac{\sqrt{13}}{3}}\right)+c$	1
		$=\frac{-1}{\sqrt{13}}\log\left(\frac{\tan\frac{x}{2}-\frac{2}{3}-\frac{\sqrt{13}}{3}}{\tan\frac{x}{2}-\frac{2}{3}+\frac{\sqrt{13}}{3}}\right)+c$	½
		$=\frac{-1}{\sqrt{13}}\log\left(\frac{3\tan\frac{x}{2}-2-\sqrt{13}}{3\tan\frac{x}{2}-2+\sqrt{13}}\right)+c$	
	c)	Evaluate : $\int\sec^3 x\,dx$	**04**
	Ans	Let $I=\int\sec^3 x\,dx$	
		$=\int\sec^2 x\cdot\sec x\,dx$	½
		$=\sec x\int\sec^2 x\,dx-\int\left[\int\sec^2 x\,dx\cdot\frac{d}{dx}\sec x\right]dx$	½
		$=\sec x\tan x-\int[\tan x\cdot\sec x\cdot\tan x]dx$	½
		$=\sec x\tan x-\int\tan^2 x\cdot\sec x dx$	
		$=\sec x\tan x-\int(\sec^2 x-1)\cdot\sec x dx$	½
		$=\sec x\tan x-\int(\sec^3 x-\sec x)dx$	
		$=\sec x\tan x-\int\sec^3 x dx+\int\sec x dx$	½
		$I=\sec x\tan x-I+\log(\sec x+\tan x)+c$	1
		$\therefore 2I=\sec x\tan x+\log(\sec x+\tan x)+c$	
		$\therefore I=\frac{1}{2}(\sec x\tan x+\log(\sec x+\tan x))+c$	½
	d)	Evaluate $\int\frac{2x^2+5}{(x-1)(x+2)(x+3)}dx$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)Ans	$\int \frac{2x^2+5}{(x-1)(x+2)(x+3)}dx$	
		Consider $\frac{2x^2+5}{(x-1)(x+2)(x+3)} = \frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x+3}$	½
		$\therefore 2x^2+5 = A(x+2)(x+3)+B(x-1)(x+3)+C(x-1)(x+2)$	
		Put $x=1 \Rightarrow$ $2(1)^2+5 = A(1+2)(1+3)$ $\therefore A = \frac{7}{12}$	½
		Put $x=-2 \Rightarrow$ $2(-2)^2+5 = B(-2-1)(-2+3)$ $\therefore B = \frac{-13}{3}$	½
		Put $x=-3 \Rightarrow$ $2(-3)^2+5 = C(-3-1)(-3+2)$ $\therefore C = \frac{23}{4}$	½
		$\therefore \frac{2x^2+5}{(x-1)(x+2)(x+3)} = \frac{\frac{7}{12}}{x-1}+\frac{\frac{-13}{3}}{x+2}+\frac{\frac{23}{4}}{x+3}$	
		$\therefore \int \frac{2x^2+5}{(x-1)(x+2)(x+3)}dx = \int\left(\frac{\frac{7}{12}}{x-1}+\frac{\frac{-13}{3}}{x+2}+\frac{\frac{23}{4}}{x+3}\right)dx$	½
		$= \frac{7}{12}\log(x-1)-\frac{13}{3}\log(x+2)+\frac{23}{4}\log(x+3)+c$	½+½+½
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	e)	Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}}\,dx$	**04**
	Ans	Let $I = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}}\,dx$	
		$= \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\frac{\cos x}{\sin x}}}dx$	½
		$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \qquad \text{-------------------}(1)$	½
		$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$	1
		$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \qquad \text{-------------------}(2)$	½
		Add (1) and (2)	
		$I + I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$	
		$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	½
		$2I = \int_0^{\frac{\pi}{2}} 1dx$	
		$2I = \left[x\right]_0^{\frac{\pi}{2}}$	½
		$2I = \frac{\pi}{2} - 0$	
		$I = \frac{\pi}{4}$	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.		**Attempt any TWO of the following:**	**12**
	a)	Find area of the region by the parabolas, $y^2 = 9x$ and $x^2 = 9y$	**06**
	Ans	$y^2 = 9x$ ---------(1)	
		$x^2 = 9y$	
		$\therefore y = \dfrac{x^2}{9}$	
		$\therefore$ equ. $(1) \Rightarrow \left(\dfrac{x^2}{9}\right)^2 = 9x$	
		$\therefore \dfrac{x^4}{81} = 9x$	
		$\therefore x^4 = 729x$	
		$\therefore x^4 - 729x = 0$	
		$\therefore x\left(x^3 - 9^3\right) = 0$	
		$\therefore x = 0, 9$	1
		Area $A = \int_a^b \left(y_1 - y_2\right) dx$	
		$\therefore A = \int_0^9 \left(3\sqrt{x} - \dfrac{x^2}{9}\right) dx$	1
		$\therefore A = \left(\dfrac{3x^{3/2}}{\frac{3}{2}} - \dfrac{x^3}{27}\right)_0^9$	2
		$= \left(\dfrac{3(9)^{3/2}}{\frac{3}{2}} - \dfrac{9^3}{27}\right) - 0$	1
		$\therefore A = 27$	1
		--	
	b)	Attempt the following:	**06**
	(i)	Form a differential equation by eliminating arbitrary constant.If $y = A\sin x + B\cos x$	**03**
	Ans	$y = A\sin x + B\cos x$	
		$\therefore \dfrac{dy}{dx} = A\cos x - B\sin x$	1
		$\therefore \dfrac{d^2y}{dx^2} = A\left(-\sin x\right) - B\cos x$	1
		$= -\left(A\sin x + B\cos x\right) = -y$	
		$\therefore \dfrac{d^2y}{dx^2} + y = 0$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)(ii)	Solve $\left(1+x^3\right)dy-x^2ydx=0$	**03**
	Ans	$\because \left(1+x^3\right)dy-x^2ydx=0$	
		$\therefore \left(1+x^3\right)dy=x^2ydx$	
		$\therefore \frac{dy}{y}=\frac{x^2dx}{1+x^3}$	1
		$\therefore$ Solution is,	
		$\int \frac{dy}{y}=\int \frac{x^2}{1+x^3}dx$	
		$=\frac{1}{3}\int \frac{3x^2}{1+x^3}dx$	1
		$\therefore \log y=\frac{1}{3}\log\left(1+x^3\right)+c$	1
		--	
	c)	An electrical circuit containing an inductance L henries resistance R in series with an electromotive force.$E\sin\omega t$ satisfies the equation $L\frac{di}{dt}+Ri=E\sin\omega t.$ Find the value of the current at any time $t,$ if initially there is no current.	**06**
	Ans	$L\frac{di}{dt}+Ri=E\sin\omega t$	
		$\therefore \frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}\sin\omega t$	
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{R}{L}$ and $Q=\frac{E}{L}\sin\omega t$	
		$IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1
		$\therefore$ Solution is	
		$i\cdot IF=\int Q\cdot IFdt+c$	
		$i\cdot e^{\frac{R}{L}t}=\int \frac{E}{L}\sin\omega te^{\frac{R}{L}t}dt+c$	1
		$=\frac{E}{L}\int \sin\omega te^{\frac{R}{L}t}dt+c----(1)$	
		Let I=$\int \sin\omega te^{\frac{R}{L}t}dt$	
		$=\sin\omega t\int e^{\frac{R}{L}t}dt-\int\left[\int e^{\frac{R}{L}t}dt\cdot\frac{d}{dt}\sin\omega t\right]dt$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$=\sin\omega t\cdot\frac{e^{\frac{R}{L}t}}{\frac{R}{L}}-\int\frac{e^{\frac{R}{L}t}}{\frac{R}{L}}\cdot\cos\omega t\cdot\omega dt$	
		$=\frac{L}{R}\sin\omega te^{\frac{R}{L}t}-\frac{L\omega}{R}\int e^{\frac{R}{L}t}\cos\omega tdt$	
		$=\frac{L}{R}\sin\omega te^{\frac{R}{L}t}-\frac{L\omega}{R}\left[\cos\omega t\cdot\int e^{\frac{R}{L}t}dt-\int\left[\int e^{\frac{R}{L}t}dt\cdot\frac{d}{dt}\cos\omega t\right]dt\right]$	½
		$=\frac{L}{R}\sin\omega te^{\frac{R}{L}t}-\frac{L\omega}{R}\left[\cos\omega t\cdot\frac{e^{\frac{R}{L}t}}{\frac{R}{L}}-\int\left[\frac{e^{\frac{R}{L}t}}{\frac{R}{L}}\cdot(-\sin\omega t)\omega\right]dt\right]$	
		$I=\frac{L}{R}\sin\omega te^{\frac{R}{L}t}-\frac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}-\frac{\omega^2L^2}{R^2}\int e^{\frac{R}{L}t}\sin\omega tdt$	
		$I=\frac{L}{R}\sin\omega te^{\frac{R}{L}t}-\frac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}-\frac{\omega^2L^2}{R^2}I$	
		$\left(1+\frac{\omega^2L^2}{R^2}\right)I=\frac{L}{R}\sin\omega te^{\frac{R}{L}t}-\frac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}$	
		$I=\frac{R^2}{R^2+\omega^2L^2}\left[\frac{L}{R}\sin\omega te^{\frac{R}{L}t}-\frac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}\right]$	
		$\therefore$ equation (1) becomes	
		$ie^{\frac{R}{L}t}=\frac{E}{L}\frac{R^2}{R^2+\omega^2L^2}\left[\frac{L}{R}\sin\omega te^{\frac{R}{L}t}-\frac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}\right]+c$	1
		initially $i=0$	
		$\therefore$ when $t=0, i=0$	
		$\therefore 0=\frac{E}{L}\frac{R^2}{R^2+\omega^2L^2}\left[\frac{-\omega L^2}{R^2}\right]+c$	
		$\therefore 0=\frac{-E\omega L}{R^2+\omega^2L^2}+c$	
		$\therefore c=\frac{E\omega L}{R^2+\omega^2L^2}$	1
		$\therefore ie^{\frac{R}{L}t}=\frac{E}{L}\frac{R^2}{R^2+\omega^2L^2}\left[\frac{L}{R}\sin\omega te^{\frac{R}{L}t}-\frac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}\right]+\frac{E\omega L}{R^2+\omega^2L^2}$	
		$\therefore i=\frac{ER}{R^2+\omega^2L^2}\left[\sin\omega t-\frac{\omega L}{R}\cos\omega t\right]+\frac{E\omega L}{R^2+\omega^2L^2}e^{\frac{-R}{L}t}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	OR	
		$L\frac{di}{dt}+Ri=E\sin\omega t$	
		$\therefore \frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}\sin\omega t$	
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{R}{L}$ and $Q=\frac{E}{L}\sin\omega t$	
		$IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1
		$\therefore$ Solution is	
		$i\cdot IF=\int Q\cdot IF dt+c$	
		$i\cdot e^{\frac{R}{L}t}=\int\frac{E}{L}\sin\omega t e^{\frac{R}{L}t}dt+c$	1
		$=\frac{E}{L}\int\sin\omega t e^{\frac{R}{L}t}dt+c -----(1)$	
		$\left[\because \int e^{ax}\sin bx dx=\frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)\right]$	
		$\therefore \int e^{\frac{R}{L}t}\sin\omega t dt=\frac{e^{\frac{R}{L}t}}{\frac{R^2}{L^2}+\omega^2}\left[\frac{R}{L}\sin\omega t-\omega\cos\omega t\right]$	1
		$\therefore$ equation (1) becomes	
		$i\cdot e^{\frac{R}{L}t}=\frac{E}{L}\frac{e^{\frac{R}{L}t}}{\frac{R^2}{L^2}+\omega^2}\left[\frac{R}{L}\sin\omega t-\omega\cos\omega t\right]+c$	1
		$=\frac{ELe^{\frac{R}{L}t}}{R^2+\omega^2L^2}\left[\frac{R}{L}\sin\omega t-\omega\cos\omega t\right]+c$	
		initially $i=0, \therefore$ when $t=0, i=0$	
		$\therefore 0=\frac{EL}{R^2+\omega^2L^2}[-\omega]+c$	
		$\therefore c=\frac{\omega EL}{R^2+\omega^2L^2}$	1
		$\therefore i\cdot e^{\frac{R}{L}t}=\frac{ELe^{\frac{R}{L}t}}{R^2+\omega^2L^2}\left[\frac{R}{L}\sin\omega t-\omega\cos\omega t\right]+\frac{\omega EL}{R^2+\omega^2L^2}$	
		$\therefore i=\frac{EL}{R^2+\omega^2L^2}\left[\frac{R}{L}\sin\omega t-\omega\cos\omega t\right]+\frac{\omega EL}{R^2+\omega^2L^2}e^{-\frac{R}{L}t}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	OR	
		$L\frac{di}{dt}+Ri=E\sin\omega t$	
		$\therefore \frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}\sin\omega t$	
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{R}{L}$ and $Q=\frac{E}{L}\sin\omega t$	
		$IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1
		$\therefore$ Solution is	
		$i\cdot IF=\int Q\cdot IF dt+c$	
		$i\cdot e^{\frac{R}{L}t}=\int\frac{E}{L}\sin\omega t e^{\frac{R}{L}t}dt+c$	1
		$=\frac{E}{L}\int\sin\omega t e^{\frac{R}{L}t}dt+c-----(1)$	
		$\left[\because\int e^{ax}\sin bx dx=\frac{e^{ax}}{\sqrt{a^2+b^2}}\sin\left(bx-\tan^{-1}\left(\frac{b}{a}\right)\right)\right]$	
		$\therefore$ equation (1) becomes,	
		$i\cdot e^{\frac{R}{L}t}=\frac{E}{L}\frac{e^{\frac{R}{L}t}}{\sqrt{\frac{R^2}{L^2}+\omega^2}}\sin\left(\omega t-\tan^{-1}\left(\frac{\omega}{\frac{R}{L}}\right)\right)+c$	1
		$i=\frac{E}{\sqrt{R^2+L^2\omega^2}}\sin\left(\omega t-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)+ce^{\frac{-R}{L}t}$	1
		initially $i=0$	
		$\therefore$ when $t=0, i=0$	
		$\therefore 0=\frac{E}{\sqrt{R^2+L^2\omega^2}}\sin\left(-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)+c$	
		$\therefore c=\frac{-E}{\sqrt{R^2+L^2\omega^2}}\sin\left(-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)$	1
		$\therefore i=\frac{E}{\sqrt{R^2+L^2\omega^2}}\left(\sin\left(\omega t-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)-\sin\left(-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)e^{\frac{-R}{L}t}\right)$	1
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.		**Attempt any TWO of the following:**	**12**
	a)(i)	Using trapezoidal rule, evaluate the approximate value of $\int_0^4 \sqrt{x}\, dx,$ given by	
		<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$y=\sqrt{x}$</td><td>0</td><td>1</td><td>1.4142</td><td>1.7321</td><td>2</td></tr></table>	**03**
	Ans	$\int_a^b f(x)dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$	
		$a=0, b=4$ and $h=1$	
		$\therefore \int_0^4 \sqrt{x}dx = \frac{1}{2}\left[(0+2)+2(1+1.4142+1.7321)\right]$	2
		$=5.1463$	1
	a)(ii)	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using trapezoidal rule by using following data:	
		<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$y=\frac{1}{1+x^2}$</td><td>1</td><td>0.5</td><td>0.2</td><td>0.1</td><td>0.588</td><td>0.0385</td><td>0.027</td></tr></table>	**03**
	Ans	$\int_a^b f(x)dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$	
		$a=0, b=6$ and $h=1$	
		$\therefore \int_0^6 \frac{1}{1+x^2}dx = \frac{1}{2}\left[(1+0.027)+2(0.5+0.2+0.1+0.588+0.0385)\right]$	2
		$=1.94$	1
	b)	Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ by Simpson's $1/3^{rd}$ rule by taking 6 sub intervals.	**06**
	Ans	Let $y=\frac{1}{1+x^2}$ $a=0, b=1$ and $n=6$	
		$\therefore h=\frac{b-a}{n}=\frac{1-0}{6}=\frac{1}{6}$	1
		<table><tr><td>x</td><td>0</td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{2}$</td><td>$\frac{2}{3}$</td><td>$\frac{5}{6}$</td><td>1</td></tr><tr><td>$y=\frac{1}{1+x^2}$</td><td>1</td><td>$\frac{36}{37}$</td><td>$\frac{9}{10}$</td><td>$\frac{4}{5}$</td><td>$\frac{9}{13}$</td><td>$\frac{36}{61}$</td><td>$\frac{1}{2}$</td></tr></table>	2
		Using Simpson's $1/3^{rd}$ rule	
		$\int_a^b f(x)dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)	$\therefore \int_0^1 f(x)dx = \frac{1/6}{3}\left[\left(1+\frac{1}{2}\right)+4\left(\frac{36}{37}+\frac{4}{5}+\frac{36}{61}\right)+2\left(\frac{9}{10}+\frac{9}{13}\right)\right]$	2
		$\therefore \int_0^1 \frac{1}{1+x^2}dx = 0.7854$	1
		OR	
		Let $y=\frac{1}{1+x^2}$ $a=0, b=1$ and $n=6$	
		$\therefore h=\frac{b-a}{n}=\frac{1-0}{6}=\frac{1}{6}=0.1667$	1
		(table below)	2
		Using Simpson's $1/3^{rd}$ rule	
		$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$	
		$\therefore \int_0^1 f(x)dx=\frac{0.1667}{3}\left[(1+0.5)+4(0.9730+0.8+0.5901)+2(0.9+0.6922)\right]$	2
		$\int_0^1 \frac{1}{1+x^2}dx=0.7855$	1
	c)	Using Simpson's $3/8^{th}$ rule to find $\int_0^{0.6} e^{-x^2}dx$ by taking seven ordinates.	**06**
	Ans	Here $n=6$	
		$y=e^{-x^2}$ $a=0,$ $b=0.6$	
		$\therefore h=\frac{b-a}{n}=\frac{0.6-0}{6}=0.1$	1
		(table below)	2
		Using Simpson's $3/8^{th}$ rule.	
		$\int_a^b f(x)dx=\frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4...+y_{n-1})+2(y_3+y_6+...+y_{n-3})\right]$	
		$\therefore \int_0^{0.6} e^{-x^2}dx=\frac{3(0.1)}{8}\left[(1+0.6977)+3(0.99+0.9608+0.8521+0.7788)+2(0.9139)\right]$	2
		$\therefore \int_0^{0.6} e^{-x^2}dx=0.5351$	1

Table for 6 b):

x	0	0.1667	0.3334	0.5001	0.6668	0.8335	1
$y=\frac{1}{1+x^2}$	1	0.9730	0.9	0.8	0.6922	0.5901	0.5

Table for 6 c):

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$y=e^{-x^2}$	1	0.99	0.9608	0.9139	0.8521	0.7788	0.6977

WINTER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Q. No.	Sub Q. N.	Answer	Marking Scheme
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.*	