22201

21819

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any FIVE of the following: **10**

a) Define Implicit function with suitable example.

b) State whether the function $f(x)=\frac{e^{x}-e^{-x}}{2}$, is even or odd.

c) Find $\frac{dy}{dx}$; if $y=(x^{4}+2x)\cdot\sin 3x$

d) Evaluate $\int x\cdot\cos x\,dx$

e) Evaluate $\int\left[e^{2\log x}+e^{x\log 2}\right]dx$

f) Find the area under the curve $y=x^{2}$ from $x = 0$ to $x = 3$ with x–axis.

g) State Simpson's one third rule of numerical integration.

P.T.O.

Marks

2. Attempt any THREE of the following: **12**

a) If $y = f(x) = \frac{x-5}{5x-1}$, show that $f(y) = x$.

b) Find $\frac{dy}{dx}$, if $13x^2 + 2x^2 y + y^3 = 1$

c) A metal wire 40 cm long is bent to form a rectangle. Find its dimensions when area is maximum.

d) Show that radius of curvature at any point on the curve $y = a \log\left(\sec \frac{x}{a}\right)$, Where a is constant is $a \sec \frac{x}{a}$.

3. Attempt any THREE of the following: **12**

a) The slope of the curve $2y^3 = ax^2 + b$ at point (1, –1) is same as the slope of $x + y = 0$. Find a and b.

b) Find $\frac{dy}{dx}$, if $y = \sec^{-1}\left[\frac{1}{4x^3 - 3x}\right]$

c) If $x = a\cos^3\theta, y = a\sin^3\theta$ *find* $\frac{dy}{dx}$ *at* $\theta = \frac{\pi}{4}$

d) Evaluate $\int \frac{e^x(x+1)}{\cos^2(xe^x)}\,dx$

4. Attempt any THREE of the following: **12**

a) Evaluate $\int \frac{\sec^2 x\,dx}{3\tan^2 x - 2\tan x - 5}$

b) Evaluate $\int \frac{dx}{1 + \sin x + \cos x}$

c) Evaluate $\int x^2 \cos 2x\,dx$

Marks

d) Evaluate $\int_5^{10} \frac{dx}{(x-1)(x-2)}$

e) Evaluate $\int_3^7 \frac{(10-x)^2}{x^2+(10-x)^2}\,dx$

5. Attempt any TWO of the following: **12**

a) Find the area of the region included between parabola $y = x^2$ and $y = 4$.

b) Attempt the following:

(i) Verify that $y = \log x$ is a solution of differential equation

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

(ii) Solve: $\frac{dy}{dx} = e^{2x-y} + x^2 e^{-y}$

c) A particle starting with velocity 6 m/s has an acceleration $(1 - t^2)$ m/s^2. When does it first comes to rest? How far has it then travelled?

Marks

6. Attempt any TWO of the following: **12**

a) Attempt the following:

(i) Using Trapezoidal rule, calculate approximate value of $\int_3^8 \log_e x \, dx$

by using following table.

x	3	4	5	6	7	8
$y = \log_e x$	1.098	1.3863	1.6094	1.7918	1.9458	2.0794

(ii) Using Simpson's $\frac{1}{3}^{rd}$ rule, calculate the approximate value of $\int_0^4 e^x \, dx$ by using following data:

x	0	1	2	3	4
$y = e^x$	1	2.72	7.39	20.09	54.60

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Trapezoidal rule, taking $n = 4$.

Hence, obtain approximate value of π.

c) Evaluate $\int_0^{\pi/2} \sqrt{\cos x} \, dx$ using Simpson's $\frac{3}{8}$ rule with $n = 8$
